

Periodic frequencies of the cycles in 2×2 games

Bin Xu^{†,‡}, Shuang Wang^{§,‡}, Zhijian Wang^{‡*1}

[†]*Public Administration College, Zhejiang Gongshang University, Hangzhou, 310018, China*

[‡]*Experimental Social Science Laboratory, Zhejiang University, Hangzhou, 310058, China*

[§]*Chu Kochen College, Zhejiang University, Hangzhou, 310058, China*

Abstract

Evolutionary dynamics provides an iconic relationship — the periodic frequencies of a game is determined by the payoff-matrix of the game. This letter reports the first experimental evidence to confirm such a relationship in two populations random-matched 2×2 games with 12 different payoff-matrix parameters.

We find, in the different parameters games, the observed periodic frequencies of cycles are different. Interesting is that, the experimental frequencies linear positive relates to the theoretical frequencies, significantly (larger than 5 sigma). The theoretical frequencies are calculated explicitly from payoff-normalized replicator dynamics — one of the simplest evolutionary dynamics.

Keywords: frequency, period, cycle, experimental economics, normalized-payoff replicator dynamics, 2×2 game, detailed balance condition
JEL classification C72, C73, C92, D83

¹Corresponding author: wangzj@zju.edu.cn

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1. Introduction

Cycle is a significant phenomenon in social evolution. Evolutionary game theory predicts cyclic behavior as well, but such behavior has rarely been confirmed clearly in experimental games [13, 7, 11].

Only quite recently, in a continuous time laboratory environment, Casson, Friedman and Hopkins (CFH, 2012 [7]) found clear persistent cycles in population state space with the method of cycle rotation indexes (CRI). After that, with the CRI method, we found that in a traditional setting Rock-Paper-Scissors (RPS) game, cycles also exist, persist and do not dissipate [24]. For all of the cycles detected in experiments, the directions of cycles meet evolutionary dynamics very well. Therefore one can say, *qualitatively*, cycles in experiments conform to the expectations of evolutionary dynamics.

Some interesting questions now appear. How the cycles change when the parameters of game is changed? Can cycles meet evolutionary dynamics *quantitatively*?

To answer these questions, we recur to periodic *frequency* — an iconic observable in cyclic behaviors. Our motivation comes from both experimental economics and the evolutionary game theory. On one hand, the frequency of the cycles in experiment can be measured. On the other hand, the frequencies can be calculated with the dynamics equations. Then, the consistency of the two sides can be tested.

Base on the 12 experiments of 2×2 games², we detect and confirm the existence, persistence of the cycles whose directions meet evolutionary dynamics. Meanwhile, each game has its own frequency in the experiment due to its unique payoff matrix. Firstly, we find the empirical cyclic frequencies relates linearly to the theoretical frequencies significantly. The theoretical frequencies are calculated explicitly from the payoff-normalized replicator dynamics³ (PNRD) [12].

The structure of this letter is as follow. In next section, we introduce the 12 games briefly, calculate the theoretical frequencies for each of the 12 games from the PNRD equations and point out the testable hypothesis. In section 3, we test the hypothesis and report the results (1) the cycles do exist and persist in the 12 games; (2) there exists a positive correlation between

²The reasons of using the 12 experiments will be explained in section 2.1.

³The reasons of using this model will be explained in section 4.2.

the theoretical frequencies and the experimental frequencies; (3) the average radius of the cycles do not change along time as expected by PNRD. The last section includes discussion and summary. Mathematical and statistical appendixes comes at the end.

2. Experiments, Eigenvalues and Testable Hypothesis

2.1. The 12 two population 2×2 games

We use the 12 experiments with a variety of parameters in an influential published literature *Stationary Concepts for Experimental 2×2 -Games* by Selten and Chmura [16].

All the 12 experiments are of the *traditional setting* which has been widely used to investigate dynamical behaviors [9, 8, 13, 16, 2]. Specifically, There are two four-person populations in each group of the games [16]. Each group play the game for 200 rounds. The strategy set for each player in the first population X is (X_1, X_2) and the second population Y is (Y_1, Y_2) . In each round, every subject plays the game against a opponent randomly picked from the other population. The payoff matrix for each game can be generalized as Fig. 1 where a_{ij} (b_{ij}), $i, j = 1, 2$ indicates the payoff for the players in the first (second) population. The game 1~6 (each consists of 12 groups) are constant sum games and the game 7~12 (each contains of 6 groups) are nonconstant sum games having the same Nash equilibriums with the game 1~6 correspondingly.

The advantage of such experimental settings can be elaborated as follows. (1) As emphasized by the original designers of the experiments, for all the game, (2) The 12 games each has a unique mixed strategy Nash equilibrium, all belong to the most fundamental classical evolutionary dynamics models whose real part of the eigenvalues are 0 and endless cycles are expected mathematically which we should show in section 2.2. (3) There are sufficient samples (12 different controlled parameters and 108 groups), so the robustness of the results could be expected. (4) The successive 200 rounds samples are long and fit to test the evolutionary behaviors. To the best our knowledge, in laboratory two-population 2×2 games in homogenous environment⁴, these 12 games are the longest in the experimental rounds, widest in the

⁴The same experimenter, the same software, same culture of subjects pool and same payment rule.

	Y_1	Y_2
X_1	a_{11}	a_{12}
	b_{11}	b_{12}
X_2	a_{21}	a_{22}
	b_{21}	b_{22}

Figure 1: Payoff matrix of the 2×2 games.

parameters setting and the largest in the involved subjects number. With these advantages, these 12 games are supposed to be representative and fit for our investigations for social dynamics.

2.2. Theoretical frequencies of the 12 games

An evolutionary dynamics model describes the dynamics properties of a system. To obtain a theoretical expectation of the frequency, we start from PNRD [12] — an almost the oldest and the simplest model in evolutionary game theory. If a social state is supposed to be at (x, y) in which x denotes the proportion of X_1 strategy used in the population X and y denotes the proportion of Y_1 strategy used in the population Y . The velocity at (x, y) , denoted as (\dot{x}, \dot{y}) [18, 21, 3, 17], can be expressed for PNRD [12] as,

$$\dot{x} = \bar{U}_X^{-1}(U_{X_1} - \bar{U}_X)x \quad (1)$$

$$\dot{y} = \bar{U}_Y^{-1}(U_{Y_1} - \bar{U}_Y)y \quad (2)$$

in which \bar{U}_X (\bar{U}_Y) is the mean payoff of the population X (Y) and U_{X_1} (U_{Y_1}) is the payoff for the individual who uses the X_1 (Y_1) strategy in the population X (Y). According to the 2×2 payoff matrix in Fig. 1, the inner equilibrium N point, denoted as (x_N, y_N) , can be obtained; Then around N , we can get the square of eigenvalues λ expressed (see Appendix A.1 and Table 1) as

$$\lambda^2 = \frac{(a_{11} - a_{21})(a_{12} - a_{22})(b_{11} - b_{12})(b_{21} - b_{22})}{(a_{12}a_{21} - a_{11}a_{22})(b_{21}b_{12} - b_{11}b_{22})}. \quad (3)$$

Substituting each of the payoff matrix of the 12 games into Eq. (3), we find all the λ^2 are negative. Denoting the eigenvalues as $\lambda = \alpha \pm i\beta$, we have λ has only pure imaginary part (β) and the real part (α) is zero⁵. Base on β ,

⁵ Mathematically, $\beta \geq 0$. α presents the tendency of the cyclic amplitudes along time (the amplitudes of the cycles will become larger, persistent or smaller when $\alpha > 0$, $\alpha = 0$ or $\alpha < 0$). β indicates the frequency who is higher when β is larger. Periods of cycles are determined by the frequencies as $1/f$.

Table 1: Payoff matrices, equilibrium, eigenvalues and theoretical frequencies

game	a_{11}	b_{11}	a_{12}	b_{12}	a_{21}	b_{21}	a_{22}	b_{22}	x_N	y_N	λ^2	α	β	f_{theo}
1	10	8	0	18	9	9	10	8	0.91	0.09	-0.01	0	0.10	0.0161
2	9	4	0	13	6	7	8	5	0.73	0.18	-0.08	0	0.29	0.0463
3	8	6	0	14	7	7	10	4	0.91	0.27	-0.04	0	0.20	0.0320
4	7	4	0	11	5	6	9	2	0.82	0.36	-0.14	0	0.37	0.0591
5	7	2	0	9	4	5	8	1	0.73	0.36	-0.28	0	0.53	0.0841
6	7	1	1	7	3	5	8	0	0.64	0.46	-0.45	0	0.67	0.1071
7	10	12	4	22	9	9	14	8	0.91	0.09	-0.01	0	0.10	0.0155
8	9	7	3	16	6	7	11	5	0.73	0.18	-0.07	0	0.26	0.0419
9	8	9	3	17	7	7	13	4	0.91	0.27	-0.03	0	0.19	0.0297
10	7	6	2	13	5	6	11	2	0.82	0.36	-0.11	0	0.34	0.0537
11	7	4	2	11	4	5	10	1	0.73	0.36	-0.21	0	0.46	0.0734
12	7	3	3	9	3	5	10	0	0.64	0.46	-0.31	0	0.55	0.0880

the theoretical periodic frequency (f_{theo}) of a game can be calculated as

$$f_{theo} = \beta/2\pi. \quad (4)$$

The f_{theo} are shown in Table 1 and will serve as the approximate⁶ theoretical expectations in Fig. 3 to compare with observed frequencies.

2.3. Testable Hypothesis

In stochastic adaptive dynamics processes, as emphasized by Young [26], in the detailed balance condition, the transitions between any pair of the two states is symmetry. In this condition, no net transit could be found in the state space. On the contrary, if there exists asymmetry transits within states, the detailed balance symmetry is broken, then pattern of the net transit must emerge in the state space [22, 1, 19, 23]. For cyclic games, *a naive hypothesis is the net cycles can be captured by the evolutionary dynamics quantitatively.*

By counting the net cycles in experimental time sequences, in the 200 rounds/session and 108 session and 12 games data, the theoretical predictions of the PNRD fall into following testable hypothesis.

- (1) The social state will move cyclically; Directions of the cycles are clockwise.
- (2) The experimental frequency (f_{exp}) of the cycles is linear correlative to the theoretical frequency (f_{theo}). f_{exp} should be 0 when f_{theo} is zero.
- (3) The the average radius of the cycles R should not change along time.

⁶The evaluations include two approximations [7]. (1) In dynamics model, the population size is assumed to be infinite, but we use this model in experimental games with finite population size. (2) Mathematically, the expansion requires the motion very close around the rest point (NE), but we use it in full state space.

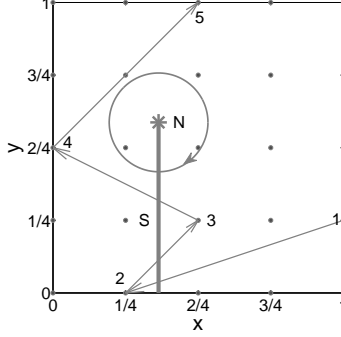


Figure 2: Schematic diagram. Background lattice represents the 25 social states. The segment S is the Poincare section from the Nash Equilibrium (N , star) to down edge. The direction of the cycle surround the center N is predicted by PNRD. An example is illustrated in Appendix A.4 for all of the measurements throughout this letter.

3. Hypothesis Tests

3.1. Test method: trajectory, Poincare section and net cycles

As mentioned, we use social state (x, y) to describes the aggregate behavior in these games [14]. As illustrated in Fig. 2, there are altogether 25 social states in a two-population society where each population contains of four persons in the games. For a certain round t , the social state must be one of the 25 points in Fig 2. Across rounds, the state (x, y) is supposed to keep transiting and form a stochastic-like trajectory [14, 7].

To verify the cycles qualitatively, we recur to the CRI (Cycle Rotation Index) method [7]. To calculate CRI, a segment (called as Poincare section [7], denoted as S , as shown in Fig. 2) is set from Nash Equilibrium N of the game to the low edge of its own state space. Every transit- i across S can be denoted as C_i and be assigned as $C_i=1$ (-1) if the transit is counterclockwise (clockwise). Then the CRI denoted by θ can be calculated as

$$\theta = \frac{\sum_i C_i}{\sum_i |C_i|}. \quad (5)$$

in which $\sum_i C_i$ is the net cycles (denoted as C_{t_0, t_1}^e) and $\sum_i |C_i|$ is the total transits across the Poincare section- S . The θ indicate the existence of cycles when it does not equals to 0. Additionally, $\theta > 0$ ($\theta < 0$) indicates the

direction of cycles is counterclockwise (clockwise)⁷. We use the subscript (t_0, t_1) to denote the observation in time interval $[t_0, t_1]$, like θ_{t_0, t_1} .

3.2. Existence, persistence and direction of the cycles

To test the existence of the net cycles in the 12 games, we calculate the average value $\bar{\theta}_{1,200}$ for the 12 games as well as the standard error over the groups. The results are shown in Table 2. Using the 12 games as samples, we find the existence of cycles is significant ($p < 0.000$, $d.f. = 11$, two tailed, t -test with $H^0 : \theta_{1,200} = 0$).

Table 2: CRI, frequency and radius of the net cycles in the 12 experiments

game	obs.	$\bar{\theta}_{1,200}$	s.e.†	p	$\bar{f}_{1,200}$	s.e.†	$\bar{R}_{1,100}$	$\bar{R}_{101,200}$	diff	p
1	12	-0.072	0.026	0.004	0.015	0.005	-0.044	-0.023	-0.022	0.003
2	12	-0.087	0.031	0.015	0.021	0.006	-0.035	-0.049	0.014	0.303
3	12	-0.141	0.044	0.009	0.014	0.004	-0.084	-0.071	-0.013	0.749
4	12	-0.129	0.018	0.003	0.031	0.005	-0.045	-0.047	0.001	0.920
5	11	-0.258	0.051	0.003	0.042	0.006	-0.079	-0.100	0.021	0.168
6	12	-0.262	0.040	0.002	0.052	0.007	-0.083	-0.065	-0.018	0.194
7	6	-0.025	0.013	0.052	0.005	0.002	-0.021	-0.010	-0.011	0.610
8	6	-0.108	0.026	0.035	0.025	0.007	-0.039	-0.073	0.034	0.129
9	6	-0.083	0.068	0.292	0.012	0.006	-0.034	-0.027	-0.007	0.841
10	6	-0.122	0.033	0.028	0.034	0.009	-0.039	-0.034	-0.005	0.505
11	6	-0.273	0.018	0.028	0.062	0.006	-0.069	-0.081	0.012	0.263
12	6	-0.222	0.063	0.028	0.045	0.012	-0.062	-0.053	-0.009	0.546

†:s.e.: standard error. ‡: Wilcoxon signed-rank test, two-tailed. Obs.: number of groups

To verify the persistence of cycles, we use Wilcoxon signed-rank test to compare the 12 game pairs values of the $\theta_{1,100}$ and $\theta_{101,200}$. Results show the null hypothesis of $H_0 : \theta_{1,100} = \theta_{101,200}$ can not be rejected ($p = 0.388$, two-tailed, $d.f. = 11$). This means the persistence of net cycles is data supported.

The empirical directions are clockwise statistically, because all $\bar{\theta}_{1,200}$ of the 12 games are negative (see Table 2). This consists the PNRD predictions (as shown in Appendix A.2).

3.3. Experimental frequency vs theoretical frequency

The experimental periodic frequency can be imaged as the time average cycles counted. Similar to Eq. (5) and references[1, 7, 23], we defined

$$f_{exp} = \frac{\sum_i C_i}{t_1 - t_0}. \quad (6)$$

⁷If we denote $C^+ = \sum_{C_i=1} |C_i|$ ($C^- = \sum_{C_i=-1} |C_i|$) to present the counterclockwise (or clockwise) transits' times. The index θ is $(C^+ - C^-)/(C^+ + C^-)$ as CRI [7].

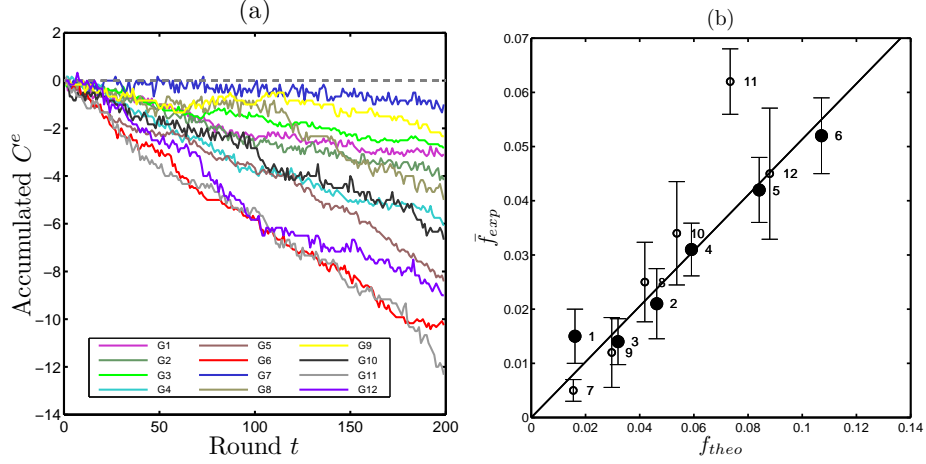


Figure 3: (a) The average accumulated $C_{1,t}^e$ of each of the 12 games. (b) Periodic frequencies of cycles. The x -axis (y -axis) is the f_{theo} (f_{exp}). The black dots indicate the mean of each 12-group game (game 1~6) and the black holes of each 6-group game (game 7 ~ 12). The bar indicates the s.e. and the numbers present the game i.d.. The slope of the line is 0.498 and the intercept is 0.002.

Fig. 3 presents net cycles number of $C_{1,t}^e$ along time t (up to 200 rounds in experiments) for the 12 games respectively. It is clear that, the mean slope of $C_{1,t}^e$ over time of these games are different. Then the frequencies are different. The frequencies of the 12 games are calculated numerically and shown in Table 2.

The relationship between the theoretical frequencies (f_{theo}) and experimental frequencies (f_{exp}) of the 12 games is demonstrated in Fig. 3. This is the main result of this letter.

Statistical results show that the coefficient of the linear regression over the 12 paired samples is 0.498 ± 0.057 . It indicates the positive correlation between theoretical frequencies and experimental frequencies is strongly significant (at $8.7\sigma > 5\sigma$ level⁸).

In addition, the constant of the regression is 0.002 ± 0.003 ($p=0.799$, two-tailed) and its 95%C.I. $[-0.005 \ 0.009]$ includes 0.000. This implicates no systematic deviation exist — f_{exp} should be zero when f_{theo} is zero.

Results from the robustness testings replicate the relationships above (at

⁸Linear regression of proportional-weighted by the proportional samples of the groups in the game. No weighted regression results shows $\sigma > 5$ also.

$\sigma > 5$ level when the center of the Poincare section set at the game mean or at the group mean. For more details, see Appendix A.3 and Table A.3.).

3.4. Radius of the net cycles

Whenever a transit- i crosses the Poincare section, the Euclidean distance (denoted as x_i) of the crossing dot to the center N can be measured (see examples in Fig. 2 and Appendix A.4). Explicitly, for a given time interval $[t_0, t_1]$, the average radius of the cycles⁹ can be defined as

$$R_{t_0, t_1} = \frac{\sum_i x_i C_i}{\sum_i |C_i|} \quad (7)$$

in which i ($=1, 2, 3, \dots$) denotes the i -th time of the trajectory passing the Poincare section, $C_i = 1$ ($C_i = -1$) if the transit is counterclockwise (clockwise); The sum covers all of the transits across the Poincare section. Results of $R_{1, 100}$ and $R_{101, 200}$ of the 12 samples are shown in Table 2. All the R values of the 12 games are negative. This also indicates the net cycles are all clockwise.

To evaluate the changing of the radius along time, we test the null hypothesis $H_0^R: R_{101, 200} = R_{1, 100}$. Using the 12 games as the 12 independent samples, we find H_0^R can not be rejected ($p = 0.965$, t -test, two-tailed, $d.f. = 11$). This indicate that the change of the radius along time, either becoming larger or smaller along time, is not supported by data.

The robustness of this result is evaluated by changing the Poincare section¹⁰, results from the 12 games samples report also the radius are of time independence. As expected by the PNRD, these 12 variety parameters 2×2 games are neutral ($\alpha = 0$) instead of stable ($\alpha < 0$, spiral in) or unstable ($\alpha > 0$, spiral out).

Till now, each of the three hypothesis in section 2.3 gains its supports from the data.

4. Discussion

In the 12 diverse 2×2 population games, we confirm the existence of the net cycles. The observed periodic frequencies of cycles are different in the

⁹This observable, as a vector, has the background like the moment of force or torque in classical mechanics in physics.

¹⁰For more details, see Table A.4 and Appendix A.3.

12 games. Interesting, the experimental frequencies linear positive relates to the theoretical frequencies which are calculated explicitly from PNRD, significantly ($> 5\sigma$). Robustness testing supports the results.

4.1. Related works and notes

The eigenvalues ($\lambda = \alpha + i\beta$) in evolutionary dynamics have been used to bridge the experimental social behaviors and evolutionary game theory [11, 6, 7]. The periodic frequencies (related to β) has also been discussed by CHF [7]. In the 12 games investigated in this letter, the imaginary parts β serves as the controlled variable and $\alpha(=0)$ serves as the invariant variable, the testing for the periodic property is possible.

On methodology view, these findings are benefitted from recently effort on experimental testing on evolutionary dynamics — transition spectrum method [25], the CRI [7], the angular momentum method [20] and the velocity field method [22]. Common point of these metrics is to capture the net transits who are of time asymmetry. In this letter, we mainly use the Poincare section method [7]. For the net cycle counting, Poincare section methos is an approximation. The results of the cycles counting could be affected by where the Poincare sections are set. In more general conditions, how to test the dynamics behavior more efficiently might be a question.

On observable view, as a novel experimental observable, periodic frequency can serve also as a testable variable for social dynamics behavior testing. This observable, which can be both theoretically calculated and experimentally measured, could be a link between evolutionary game theory and experiments.

4.2. Puzzles on the evolutionary dynamics models

In the tens and still growing-up numbers of the evolutionary dynamics models [21, 10, 14], to capture the experimental frequencies, we firstly use PNRD — an almost the oldest and the simplest model. Mainly, we consider two aspects following. (1) Replicator dynamics is the simplest model. (2) The payoff values in the dynamics equations need to be normalized, because the time has been normalized by experimental discrete time rounds. Even though the significance of their linear relationship is up to 5σ and the constant of the regression closes 0, however, the theoretical frequencies from PNRD is

twice as much as the experimental frequencies. This is a crucial puzzle¹¹.

Admittedly, we are not aware now which dynamics will perform better than PNRD among the tens dynamics models [21, 14] or the learning models [5]. Nevertheless, the results in Fig. 3 provide a hint that evolutionary game theory can be expected to be tested in laboratory experiments.

4.3. Summary

Using the 12 different parameters 2×2 games, besides confirm the observations of the cycles in experimental games [7], we are interested to find the experimental periodic frequencies of the cycles are positively relative to the theoretical periodic frequencies significant ($>5\sigma$ level). The theoretical periodic frequencies are calculated from the payoff-normalized replicator dynamics — an almost the oldest and the simplest model.

Appendix A.

Appendix A.1. A Mathematical derivation for the eigenvalues

To give a mathematical derivation of the eigenvalues, we start from the PNRD equation. The equations are specified by the 2×2 payoff matrix as Table 1. Symbols in this section are elaborated in reference [15].

Suppose x (y) is the proportion of the first population using strategy X_1 (Y_1), it is natural that $X_2 = 1 - X_1$ and $Y_2 = 1 - Y_1$. Then the average payoff for the four strategies can be expressed as the following matrix.

$$\begin{pmatrix} U_{X_1} \\ U_{X_2} \\ U_{Y_1} \\ U_{Y_2} \end{pmatrix} = \begin{pmatrix} a_{11}y + a_{12}(1-y) \\ a_{21}y + a_{22}(1-y) \\ b_{11}x + b_{21}(1-x) \\ b_{12}x + b_{22}(1-x) \end{pmatrix}$$

where U_{X_i} (U_{Y_i}), $i = 1, 2$ is the payoff for a player in the first population choosing strategy X_i (Y_i). Further, the average payoff for each population can be aggregated as,

$$\begin{pmatrix} \bar{U}_X \\ \bar{U}_Y \end{pmatrix} = \begin{pmatrix} xU_{X_1} + (1-x)U_{X_2} \\ yU_{Y_1} + (1-y)U_{Y_2} \end{pmatrix}$$

¹¹It is crucial when comparing with the condition in general physics. For example, in an RCL-circuit, the theoretical resonant frequencies from the dynamics equations can predict the experimental frequency exactly.

According to the payoff-normalized replicator dynamics (PNRD) equation [12], we obtain the a system of ordinary differential equations (ODE)

$$\begin{aligned}\dot{x} &= x(U_{X_1} - \bar{U}_X)\bar{U}_X^{-1} \\ \dot{y} &= y(U_{Y_1} - \bar{U}_Y)\bar{U}_Y^{-1}.\end{aligned}$$

Specified for 2×2 games whose payoff matrix are shown as Fig. 1, the ODE above can be presented as

$$\dot{x} = \frac{x(1-x)[(a_{12} - a_{22}) - (a_{12} - a_{22} + a_{21} - a_{11})y]}{(a_{11} - a_{21} + a_{22} - a_{12})xy + (a_{12} - a_{22})x + (a_{21} - a_{22})y + a_{22}} \quad (\text{A.1})$$

$$\dot{y} = \frac{y(1-y)[(b_{21} - b_{22}) - (b_{21} - b_{22} + b_{12} - b_{11})x]}{(b_{11} - b_{12} + b_{22} - b_{21})xy + (b_{21} - b_{22})y + (b_{12} - b_{22})x + b_{22}} \quad (\text{A.2})$$

Apparently, there exists a unique inner Nash equilibrium N in the rectangular coordinate system constructed by x and y ($0 \leq x, y \leq 1$), which is

$$\left(\frac{b_{21} - b_{22}}{b_{21} - b_{22} + b_{12} - b_{11}}, \frac{a_{12} - a_{22}}{a_{12} - a_{22} + a_{21} - a_{11}} \right). \quad (\text{A.3})$$

Then surrounding the equilibrium point N , the Jacobian of Eq. (A.1) is

$$J = \begin{pmatrix} 0 & \frac{(b_{11}-b_{12})(b_{21}-b_{22})(a_{12}-a_{22}+a_{21}-a_{11})^2}{(a_{12}a_{21}-a_{11}a_{22})(b_{21}-b_{22}+b_{12}-b_{11})^2} \\ \frac{(a_{11}-a_{21})(a_{12}-a_{22})(b_{21}-b_{22}+b_{12}-b_{11})^2}{(b_{21}b_{12}-b_{11}b_{22})(a_{12}-a_{22}+a_{21}-a_{11})^2} & 0 \end{pmatrix}.$$

Then, the square of the eigenvalues of Jacobian is

$$\lambda^2 = \frac{(a_{11} - a_{21})(a_{12} - a_{22})(b_{11} - b_{12})(b_{21} - b_{22})}{(a_{12}a_{21} - a_{11}a_{22})(b_{21}b_{12} - b_{11}b_{22})} \quad (\text{A.4})$$

which is the Eq.(3) in main text.

Substituting the payoff matrices of the 12 games into Eq.(3) in main text, we find that their eigenvalues squared are all negative real numbers as λ^2 column in Table 1. Denoting λ as $\alpha + i\beta$, one can notice that, the real part of the eigenvalue α equals 0 and only the imaginary part left.

This means the eigenvalues of the 12 games are the pure imaginary values, even though the values differ for each of the 12 games. Then, the theoretical periodic frequencies in Eq.(4) should be different for the 12 games.

Appendix A.2. Theoretical directions of the cycles

We use PNRD equations to demonstrate the directions of the cycles of the 12 games respectively. All of the velocity vector fields are cyclic and clockwise as shown in Fig. A.4.

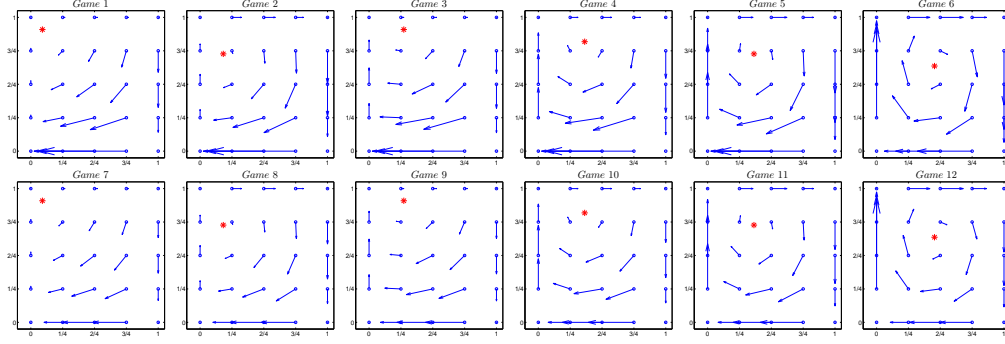


Figure A.4: PNRD expectation of the transit directions of the 12 games.

Table A.3: OLR results of f_{exp} vs f_{theo} in various Poincare sections

Poincare section	f_{exp}	$Coeff.$	$Std.Err.$	t	$P > t $	[95% C.I.]	
Nash equilibrium	f_{theo}	0.4983109	0.0572828	8.70	0.000	0.3706768	0.6259450
	$cons$	0.0021428	0.0032894	0.65	0.529	-0.0051864	0.0094719
Observed average of 12(6) observations	f_{theo}	0.4110736	0.0762773	5.39	0.000	0.2411172	0.5810299
	$cons$	0.0115081	0.0059308	1.94	0.081	-0.0017065	0.0247227
Observed average of each group's observations	f_{theo}	0.390514	0.0691754	5.65	0.000	0.2363816	0.5446464
	$cons$	0.0152928	0.0052301	2.92	0.015	0.0036395	0.0269461

Table A.4: Changing of R along time in various Poincare sections

game	obs.	Game mean			Session mean		
		$R_{1,100}$	$R_{101,200}$	p	$\bar{R}_{1,100}$	$\bar{R}_{101,200}$	p
1	12	0.043	0.032	0.315	0.049	0.027	0.179
2	12	0.027	0.046	0.151	0.042	0.052	0.551
3	12	0.034	0.046	0.452	0.039	0.061	0.362
4	12	0.071	0.078	0.786	0.055	0.074	0.233
5	11	0.09	0.099	0.576	0.076	0.091	0.165
6	12	0.075	0.06	0.324	0.074	0.06	0.243
7	6	0.032	0.017	0.594	0.031	0.014	0.52
8	6	0.042	0.095	0.063	0.049	0.077	0.168
9	6	0.03	0.014	0.534	0.045	0.033	0.616
10	6	0.038	0.032	0.579	0.055	0.045	0.573
11	6	0.064	0.079	0.301	0.068	0.072	0.83
12	6	0.051	0.05	0.914	0.056	0.056	0.992

Appendix A.3. Robustness testing on changing Poincare section

The Poincare section method suggested by CFH [7] is efficient for net cycles detection. However, where the Poincare section should be set is a puzzle. Because it is not clear where the center of cycles is in the stochastic trajectory and different learning models provides different centers [16, 4]. So, the robustness of above results need to be tested.

To test the robustness of the results on frequencies and radius of cycles, we use three various settings of Poincare section center (1) Nash equilibrium (reported in the main text), (2) observed average of observations of each the 12 games and (3) observed average of observations of each of the 108 groups, respectively.

On the relationships of f_{exp} the f_{theo} — The regression results are exhibited in Table A.3. The positive correlation relationship between f_{exp} and f_{theo} are established in all three situations. Additional testings show that the positive correlation relationships are significant (at $\sigma > 5$ level with 0-constant). The ordinary linear regression are proportional-weighted by the number of groups of the games. Such results state clearly that the relationship between experimental periodic frequencies and theoretical periodic frequencies do not dependent on the Poincare section selecting. Meanwhile, the results also announce that the Nash equilibrium performs better than the other two situations.

On the changing of the cycle radius R — The $R_{1,100}$ and $R_{101,200}$ of each game (and also the results of paired sample t -test over each groups) are exhibited in Table A.4. R is not changing when the Poincare section center is changed to game mean center ($p=0.473$, t -test, two-tails, $d.f.=11$) or group mean center ($p=0.7018$, t -test, two-tails, $d.f.=11$).

Appendix A.4. An example of the measurements

In the 12 experiments, each group play the game for 200 rounds, so the trajectory has 200 nodes in state space and 199 transits are observed. To make the concepts used in this letter easier to understand, we suppose an "experimental observed" chronological stochastic trajectory to demonstrate the concepts and the measurements.

In the example, the Poincare section begins from (0.3636, 0.5873) in the state space as Fig. 2 in main text illustrates. The hypothetical trajectory has 4 transits (transit-1, transit-2, transit-3 and transit-4) and the order of $1 \rightarrow 2 \rightarrow \dots \rightarrow 5$. The time interval is [1, 5].

Specifically, on this "experimental observed" trajectory, the social state of 3rd round is at $(\frac{2}{4}, \frac{1}{4})$, which means 2 (1) subjects use X_1 (Y_1) and 2 (3) subjects use X_2 (Y_2) in population X (Y) respectively.

From 1st to 2nd round, the transit-1 passes the S and its C_1 -value in Eq. (5) is -1 and x_1 -value in Eq (7) is 0.549; From 2nd to 3rd round, the transit-2 passes the S . Its C_2 -value in Eq. (5) is 1 and x_2 -value in Eq. (7) is 0.474.

According to Eq. (5), CRI (Cycle Rotation Index) $\theta_{1,5}$ is -0.333.

According to Eq. (6), the frequency f_{exp} is 0.250.

According to Eq. (7), the radius of the "net cycles" ($R_{1,5}$) is -0.115.

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